Application of the Quasi-estimation and its Comparison with Some Other Methods

A. Gordinsky, PhD, Berman Engineering Ltd

The essence of the quasi-estimation consists in the following. For the well-known linear gaussian regression $Y = X\beta + \varepsilon$ we use instead OLS-estimator two alternative nonlinear estimators:

$$b_1 = b + \sqrt{\frac{\lambda}{\pi}\frac{\Gamma((n-k+1)/2)}{\Gamma((n-k+2)/2)}}\sqrt{e^T e}\; z \quad \text{and} \quad b_2 = b - \sqrt{\frac{\lambda}{\pi}\frac{\Gamma((n-k+1)/2)}{\Gamma((n-k+2)/2)}}\sqrt{e^T e}\; z .$$

In these equations: b is LS-estimator, e is the regression residual, z is the eigenvector of matrix $(X^T X)^{-1}$ corresponding to maximal eigenvalue λ , Γ is gamma-function, n ,k are numbers of observations and regressors. It is proved that one of these two estimators (b_1 or b_2) have considerably smaller expectation of the square of its distance to unknown parameter β. We should use additional external information to choose the better estimate.

Considered are the following examples of use of the additional information. 1. Case of confirmation of one of the competing theories by the experiment and its outcome statistical processing. The data of checking by astronomers Dyson and others of Einstein's generalized relativity theory are analyzed, and the advantage of the quasi - estimation is shown.
2. Case of a known sign at least of one component of the vector β; that usually takes place for systems with feedback. 3. A knowledge of some features of the exact response $X\beta$ (standard deviation, bounds etc.). 4. A knowledge of the inequalities for at least one component of the vector β; in this case the quasi – estimation have an advantage not only comparably with OLS-estimator but with LS-estimator with the restriction in the form of the inequalities.

Comparably with the shrinkage estimation, including the ridge regression, the quasi-estimation has an own niche. This is the one-dimensional regression and the regression with k=2 - 4, orthogonal or non-orthogonal, with or without a multicollinearity. As to ridge regression and some other methods proposed technique does not require to find parameter dependent on the unknown β.Additionally we emphasize the following fact. An ideology of the shrinkage estimation is based on the assumption of increase of the vector b norm, especially in the case of multicollinearity. We show it cannot be observed at least for the regression of the mild dimension.